

II * M129 Summer Course 2014-2015

① Sets Numbers

$$N = \text{Natural} = \{1, 2, 3, 4, \dots\}$$

$$W = \text{Whole} = \{0, 1, 2, 3, \dots\}$$

$$Z = \text{Integers} = \{0, \pm 1, \pm 2, \dots\}$$

$$Q = \text{Rational} = \left\{ \frac{a}{b}, a, b \in Z, b \neq 0 \right\}$$

0.321 = $\frac{321}{1000}$ = 0.4

$$I = \text{Irrational} = \text{قوة غير متناهية جزو}$$

$$R = \text{All Numbers} \quad \sqrt{5}, \sqrt{3}, \frac{22}{7}, \dots$$

② Domain :-

① Rational function

$$f(x) = \frac{h(x)}{g(x)}$$

$$\Rightarrow \text{domain} = R - \{ \text{zero's of } g(x) \}$$

Ex: ① $\frac{2x}{3x-6}$

$$3x-6 \neq 0 \Rightarrow \text{domain } R - \{2\}$$

Ex: ② $\frac{x-3}{x^2+4} = f(x)$

$$x^2+4 \neq 0 \Rightarrow \text{domain} = R$$

② Root function

$$f(x) = \sqrt{g(x)}$$

$$\text{domain} = \text{intervals of } g(x) \geq 0$$

Ex $f(x) = \sqrt{3-x}$

$$\begin{array}{r} 3 \\ -1 \\ \hline 4 \end{array}$$

$$[-\infty, 3]$$

Ex $\sqrt{x^2-16}$

$$\begin{array}{r} -4 \quad 4 \\ -1 \quad -1 \\ \hline 16 \end{array}$$

$$(-\infty, -4] \cup [4, \infty)$$

③ Polynomial

$$f(x) = x^2 + 5x - 7$$

$$g(x) = (x-1)^2$$

$$\text{domain} = R$$

② Zeros of any function : The value in which = 0

Ex 1 $f(x) = \frac{2x-6}{x^2-16}$

Zeros

$$2x-6=0$$

$$x=3$$

domain

$$x^2-16 \neq 0$$

$$x \neq \pm 4$$

$$R \setminus \{ \pm 4 \}$$

Ex 2

$$f(x) = x^2 + 2x - 3$$

$$\text{Zeros } x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x=1$$

$$x=-3$$

ف

نقاط التقاطع

③ The Points of intersection: $\begin{cases} y = \dots \\ y = \dots \end{cases}$

معاداة

Ex $\begin{cases} y = x^2 + 5x \\ y = 2x^2 + 6 \end{cases} \Rightarrow x^2 + 5x = 2x^2 + 6 \Rightarrow x^2 - 5x + 6 = 0$

$x = 2$

$x = 3$

Points (2, 14)

(3, 24)

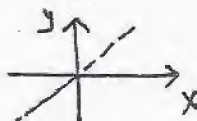
لغوض في
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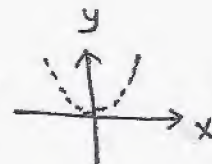
* Some Skills:

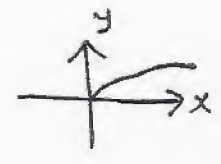
① $(a+b)^2 = a^2 + 2ab + b^2$

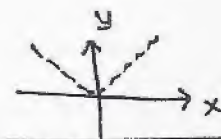
② $a^2 - b^2 = (a-b)(a+b)$

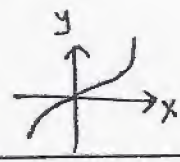
③ Some graphs

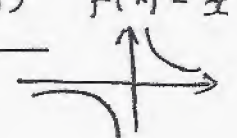
(a) $f(x) = x$ 

(b) $f(x) = x^2$ 

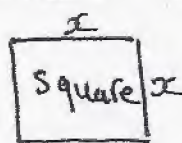
(c) $f(x) = \sqrt{x}$ 

(d) $f(x) = |x|$ 

(e) $f(x) = x^3$ 

(f) $f(x) = \frac{1}{x}$ 

④ shape



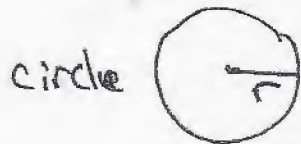
Perimeter
 $4x$

Area
 x^2



$2x + 2y$

$x \cdot y$



$2\pi r$

πr^2



Surface
 $6x^2$

Volume
 x^3

Rectangular



$2x^2 + 4x \cdot y$

$x^2 \cdot y$

انتبه:-
(حذف وجهين بدون غطاء)
أو بدون واجهة

$2x^2 + 2xy = \text{Canvas}$ لو

$2x^2 + 3xy$ بدون غطاء علوي
أو واجهة

2201

□ operation of function:

$$+ \quad \circ \quad - \quad \circ \quad \times \quad \circ \quad \div \quad \circ \quad 0$$

Ex: $f(x) = x^2 + 6x$
 $g(x) = x + 3$

$$* f \cdot g = x^3 + 9x^2 + 18x$$

$$* f + g = x^2 + 7x + 3$$

$$* f - g = x^2 + 5x - 3$$

$$* \frac{f}{g} = \frac{x^2 + 6x}{x + 3} \quad x \neq -3$$

$$* f \circ g(x) = (x+3)^2 + 6(x+3)$$

$$f[g(x)] = x^2 + 6x + 9 + 6x + 18$$

$$= x^2 + 12x + 27$$

$$\left. \begin{aligned} g \circ f(x) &= \\ g[f(x)] &= (x^2 + 6x) + 3 \\ &= x^2 + 6x + 3 \end{aligned} \right\}$$

Ex: $f(x) = x^2 + 5$, $g(x) = \sqrt{6-x}$
 $\text{domain } f = D_f = \mathbb{R}$ $D_g = (-\infty, 6]$

$$f \circ g(x) = (\sqrt{6-x})^2 + 5 = 11 - x$$

$$D_{f \circ g} = (-\infty, 6]$$

$$\left. \begin{aligned} g \circ f(x) &= \sqrt{6 - (x^2 + 5)} \\ &= \sqrt{1 - x^2} \\ D_{g \circ f} &= [-1, 1] \end{aligned} \right\}$$

Slope : m

(a)

$m = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$
 When we have two points $(x_1, y_1), (x_2, y_2)$

$\boxed{y - y_1 = m(x - x_1)}$ Equation of Line (tangent)

(b)

equation $y = mx + b$, Point (x_1, y_1)

$\Rightarrow (y - y_1) = m(x - x_1)$ (Parallel)
 متوازيان

$\perp (y - y_1) = -\frac{1}{m}(x - x_1)$

(Perpendicular)
 متعامدان

وسوف نتعامل مع المنحنيات في المنحنيات لتعارف

Where Stationary Point \iff Critical Point
 $\iff f'(x) = 0$

Derivatives:

- ① $f(x) = A$ constant $\Rightarrow f'(x) = 0$
- ② $f(x) = x^n \Rightarrow f'(x) = n x^{n-1}$
- ③ $f(x) = [g(x)]^n \Rightarrow f'(x) = n [g(x)]^{n-1} \cdot g'(x)$

Rules

Ex:

① $f(x) = x^3 + 2x^2 - 5x + 7 \Rightarrow f'(x) = 3x^2 + 4x - 5$

② $f(x) = (x^2 - 5x)^7 \Rightarrow f'(x) = 7(x^2 - 5x)^6 \cdot (2x - 5)$

③ $f(x) = \frac{1}{x^2} + x^2 + 3x \Rightarrow$ نعدل في صورتها

$f(x) = x^{-2} + x^2 + 3x \Rightarrow f'(x) = -2x^{-3} + 2x + 3$

④ $f(x) = \frac{1}{\sqrt{x^2 - x}} + 3x^2 - 4$

$f(x) = (x^2 - x)^{-\frac{1}{2}} + 3x^2 - 4 \Rightarrow f'(x) = -\frac{1}{2}(x^2 - x)^{-\frac{3}{2}} \cdot (2x - 1) + 6x$

* Derivatives by using The definition: - * أنواع ص ١

(a)

$f(x) = x^2 - 2x$

Solution

* $f(x+h) - f(x) =$

$= (x+h)^2 - 2(x+h) - (x^2 - 2x)$

$= x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x$

$= 2xh + h^2 - 2h$

$= h(2x + h - 2)$ عامل مشترك

* نضع ما وصلنا إليه ÷ h

$\frac{h(2x + h - 2)}{h}$

$= 2x - 2$

(b)

$f(x) = \frac{1}{x-3}$

Solution

* $f(x+h) - f(x) =$

$= \frac{1}{x+h-3} - \frac{1}{x-3}$

$= \frac{x-3 - (x+h-3)}{(x+h-3)(x-3)}$

$= \frac{-h}{(x+h-3)(x-3)}$

* نضع ما وصلنا إليه ÷ h

$= \frac{-1}{(x-3)(x-3)} = \frac{-1}{(x-3)^2}$

(c) خطوات

$f(x) = \sqrt{x+3}$

Solution

* $f(x+h) - f(x) =$

$= \sqrt{x+h+3} - \sqrt{x+3}$

$= \frac{\sqrt{x+h+3} - \sqrt{x+3}}{1} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}}$

$= \frac{x+h+3 - x-3}{\sqrt{x+h+3} + \sqrt{x+3}}$

$= \frac{h}{\sqrt{x+h+3} + \sqrt{x+3}}$

$= \frac{1}{2\sqrt{x+3}}$



Ex:

Solution



$$y - y_1 = m(x - x_1)$$

$$y + 1 = -2(x + 2)$$

$$y = -2x - 5$$

Ex.

$$f(x) = (3x+1)^5$$

Solution:

$$P'(x) = 5(3x+1)^4 \cdot 3 = 15(3x+1)^4$$

$$f''(x) = 15 \times 4 (3x+1)^3 \cdot 3$$
$$= 180 (3x+1)^3$$

* Position - Velocity - Acceleration
المكان السرعة التسارع

$$S(t) \xrightarrow{\frac{d}{dt}} V(t) \xrightarrow{\frac{d}{dt}} Q(t)$$

لثقت المائة مرة = المربعة وثبتت مرة = الثار

* Ex:

$$v(t) = 40 - 10t \quad \text{m/s}$$

$$a(t) = -10 \quad \text{m/s}^2$$

Find The velocity
at $t = 2$

$$v(2) = 40 - 20(2)$$

$$= 40 - 20$$

$$= 20 \text{ m/s}$$

When Hits The ground

$$s(t) = 40t - 5t^2 + 100$$

$t = -2$ و $t = 10$ = 0

Find The Hieght distance

$$v(t) = 0 \Rightarrow 40 - 10t = 0$$

$$t = 4$$

$$S(4) = 40(4) - 5(4)^2 + 100$$

$$= 180 \text{ meter}$$



6] Note x-intercept put $y=0$
y-intercept put $x=0$

Ex: $y = x^2 - 3x - 4$ ———— x-intercept $y=0$
 $\Rightarrow x^2 - 3x - 4 = 0 \Rightarrow x_1 = 4$
 $x_2 = -1$
y-intercept $x=0$
 $\Rightarrow y = 0 - 0 - 4 = -4$

2] Number of stationary points (or critical)
نقاط الـ stationery ($f'(x) = 0$)

Ex: $f(x) = x^5 - 4x^4$

Solution

$$f'(x) = 5x^4 - 16x^3 = x^3(5x - 16) = 0$$

$$\begin{array}{c|c} x^3 = 0 & 5x - 16 = 0 \\ \hline x_1 = 0 & x_2 = 16/5 \end{array}$$

Number = 2 \therefore لدينا اثنان نقطتين

Application of derivatives:

$f'(x) = 0$ Interval increasing-decreasing تزايد-تناقص
Local Maximum - Local Minimum عظمى صغرى محلية
Points

$f''(x) = 0$ Interval concave up-down تقعول-اسفل-اعلى
inflection Points نقطة انقلاب

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Ex 1

$$\text{Let } f(x) = x^3 + 3x^2 - 24x + 6$$

- a) Find $f'(x)$ and $f''(x)$.

$$f(x) = x^3 + 3x^2 - 24x + 6$$

$$f'(x) = 3x^2 + 6x - 24,$$

$$f''(x) = 6x + 6$$

- b) Find the intervals on which $f(x)$ is increasing or decreasing.

$$f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = (x + 4)(x - 2), x = -4 \text{ or } x = 2$$

intervals	$(-\infty, -4)$	$(-4, 2)$	$(2, \infty)$
$f'(x)$	+++++	-----	+++++
Conclusion	Increasing	decreasing	increasing

- c) Find the local maximum and minimum of $f(x)$, if any.
 local max $(-4, 86)$
 Local min $(2, -22)$
- d) Find the intervals on which the graph of $f(x)$ is concave up or concave down.

$$f''(x) = 6x + 6 = 6(x + 1), x = -1$$

intervals	$(-\infty, -1)$	$(-1, \infty)$
$f''(x)$	-----	+++++
Conclusion	Concave down	Concave up

Inflection point $(-1, 32)$

Ex 2

$$\text{Let } f(x) = x^4 + 4x^3 + 2$$

- a) Find $f'(x)$ and $f''(x)$

$$f(x) = x^4 + 4x^3 + 2$$

$$f'(x) = 4x^3 + 12x^2,$$

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$$f''(x) = 12x^2 + 24x$$

b) Find the intervals on which $f(x)$ is increasing or decreasing.

$$f'(x) = 4x^3 + 12x^2 = 4x^2(x + 3) = 0, x = 0 \text{ or } x = -3$$

intervals	$(-\infty, -3)$	$(-3, 0)$	$(0, \infty)$
$f'(x)$	-----	+++++	+++++
Conclusion	decreasing	increasing	increasing

c) Find the local maximum and minimum of $f(x)$, if any.

No local max

Local min $(-3, -25)$

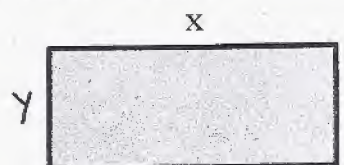
d) Find the intervals on which the graph of $f(x)$ is concave up or concave down.

$$f''(x) = 12x^2 + 24x = 12x(x + 2) = 0, x = -2 \text{ or } x = 0$$

intervals	$(-\infty, -2)$	$(-2, 0)$	$(0, \infty)$
$f''(x)$	+++++	-----	+++++
Conclusion	Concave up	Concave down	Concave up

Ex:1

A carpenter is building a rectangular room with a fixed perimeter of 360 ft. What are the dimensions of the largest room that can be built? What is its area?



$$2x + 2y = 360, y = 180 - x, A = x \cdot y,$$

$$A = x(180 - x) = 180x - x^2,$$

$$A' = 180 - 2x, 180 - 2x = 0, x = 90, y = 90$$

$$\text{Area} = 90 * 90 = 8100 \text{ ft}^2$$

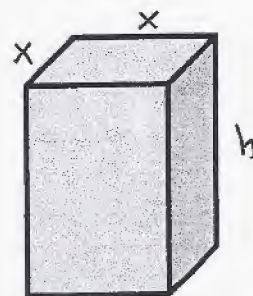
Ex 2

Find two positive numbers such that their product is 75, and so that the first plus three times the second is a minimum.

$$\begin{aligned}
 xy &= 75, & y &= \frac{75}{x} \\
 s &= x + 3y, \\
 s &= x + 3 \cdot \frac{75}{x} \\
 s &= x + \frac{225}{x}, \\
 s' &= 1 - \frac{225}{x^2} = \\
 \frac{x^2 - 225}{x^2} &= 0, \\
 x &= \pm 15, \text{ so } x = 15, & y &= 5
 \end{aligned}$$

Ex 3

Consider rectangular box with a square base. Find the dimensions of the box for which the volume is 1000 cubic feet and the total surface area for the box is minimal.



$$S = 2x^2 + 4xh, \quad \text{subject to } 1000 = x^2h$$

$$\Rightarrow S = 2x^2 + 4x \cdot \frac{1000}{x^2} = 2x^2 + \frac{4000}{x}.$$

$$S' = 4x - \frac{4000}{x^2} = 0 \Rightarrow 4x^3 = 4000 \Rightarrow x = 10, h = 10,$$

Dimensions : 10feet \times 10feet \times 10feet.

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* chapter (3) (4) (5)

* Rules of derivatives

① Product Rule

$$: (f \cdot g)' = f \cdot g' + g \cdot f'$$

② Quotient :

$$\left(\frac{f}{g} \right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \quad g \neq 0$$

③ Root : $\left(\sqrt{f(x)} \right)' = \frac{f'(x)}{2 \sqrt{f(x)}}$

④ Chain Rule :

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

⑤ Implicit Diff.

as we know $\frac{d}{dx} x^2 = 2x$

$$\frac{d}{dx} y^2 = 2y \frac{dy}{dx} = 2y y'$$

(ضع عند مشتقها بجوابها)

$$\begin{aligned} \frac{d}{dx} e^{f(x)} &= f'(x) \cdot e^{f(x)} \\ \frac{d}{dx} a^{f(x)} &= f'(x) \cdot a^{f(x)} \cdot \ln a \end{aligned}$$

$$e = 2.7182...$$

[Properties of Exponential Function] $\begin{cases} a^x \cdot a^y = a^{x+y} \\ a^x \div a^y = a^{x-y} \\ (a^x)^y = a^{x \cdot y} \end{cases}, a^0 = 1, a \neq 0$

III

* Examples

Find $f'(x)$

① $f(x) = (2x+3)(x^2-4x)$

$$f'(x) = (2x+3)(2x-4) + (x^2-4x)(2)$$

$$= 4x^2 - 2x - 12 + 2x^2 - 8x = 6x^2 - 10x - 12$$

② $f(x) = \frac{3x-4}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(3) - (3x-4)(2x)}{(x^2+1)^2} = \frac{3x^2+3-6x^2+8x}{(x^2+1)^2}$$

$$= \frac{-3x^2+8x+3}{(x^2+1)^2}$$

③ $f(x) = \sqrt{x^2+5x} \Rightarrow f'(x) = \frac{2x+5}{2\sqrt{x^2+5x}}$

④ $f(x) = \frac{(x+1)}{\sqrt{x^2+5}}$ $\xrightarrow{\text{حولها}} (x+1)(x^2+5)^{-\frac{1}{2}}$

تفاعل معها
حاصل ضرب

$$f'(x) = (x+1) \cdot \frac{1}{2} \cdot 2x(x^2+5)^{-\frac{3}{2}} + (x^2+5)^{-\frac{1}{2}} \cdot 1$$

$$= \frac{-x(x+1)}{(x^2+5)^{\frac{3}{2}}} + \frac{1}{\sqrt{x^2+5}}$$

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⑤ IF $y = \sqrt{u+1}$, $u = 2x^2 \Rightarrow$ Find $\frac{dy}{dx}$

$$\Rightarrow y = \sqrt{u+1} = \sqrt{2x^2+1}$$

$$\Rightarrow y' = \frac{dy}{dx} = \frac{4x}{2\sqrt{2x^2+1}} = \frac{2x}{\sqrt{2x^2+1}}$$

⑥ $4y^3 - x^2 = -5$ Use Implicit diff. to determine The slope at The Point (3,1)

Solution

(a) Find y' ?

$$\Rightarrow 12y^2 y' - 2x = 0$$

$$\Rightarrow y' = \frac{2x}{12y^2} = \frac{x}{6y^2}$$

$$\text{slope} = y' \Big|_{\substack{x=3 \\ y=1}} = \frac{3}{6(1)^2} = \frac{3}{6} = \frac{1}{2}$$

⑦ Find The tangent equation of $\sqrt{x} + \sqrt{y} = 7$ at (16,9)

Solution

(a) Find slope = m

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} y' = 0$$

$$\Rightarrow \frac{1}{2\sqrt{y}} y' = -\frac{1}{2\sqrt{x}}$$

$$\Rightarrow y' = \frac{-\sqrt{y}}{\sqrt{x}} \Big|_{x=16} = -\frac{3}{4}$$

(b) Find equation $(y-y_1) = m(x-x_1)$

$$\Rightarrow y - 9 = -\frac{3}{4}(x - 16)$$

$$y = -\frac{3}{4}x + 13$$

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⑧

Solution

$$f(x) = (x^2 - 5x) \cdot e^{3x}$$

$$f'(x) = (x^2 - 5x) \cdot 3 \cdot e^{3x} + e^{3x} \cdot (2x - 5)$$

$$= e^{3x} [3x^2 - 15x + 2x - 5]$$

$$= e^{3x} [3x^2 - 13x - 5]$$

⑨

$$y = \frac{e^x}{x + e^x}$$

Find

(a) y'

(b) tangent line at $(0, 1)$

Solution

(a)
فر

$$y' = \frac{(x + e^x) \cdot e^x - e^x (1 + e^x)}{(x + e^x)^2} = \frac{x e^x + e^{2x} - e^x - e^{2x}}{(x + e^x)^2}$$

$$= \frac{x \cdot e^x - e^x}{(x + e^x)^2} \Big|_{x=0} = m = \text{slope} = \frac{0 - 1}{(0 + 1)^2} = -1$$

(b) equation

$$(y - 1) = -1(x - 0)$$

$$\Rightarrow y = -x + 1$$

⑩

$$x^2 y^2 + 3x^2 = 5y$$

Find y' (Implicit)

Solution

$$(x^2 \cdot 2yy' + y^2 \cdot 2x) + 6x = 5yy'$$

$$\Rightarrow 2x^2yy' - 5yy' = -6x - 2xy^2$$

$$\Rightarrow y'(2x^2y - 5y) = -6x - 2xy^2$$

$$\Rightarrow y' = \frac{-6x - 2xy^2}{2x^2y - 5y}$$

#

14

* Logarithm function :-

Properties: 1- $\ln(xy) = \ln x + \ln y$.

2- $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$.

3- $\ln x^n = n \ln x$.

4- $\ln e^{f(x)} = e^{\ln f(x)} = f(x)$.

5- $\ln e = 1$

Ex(1) simplify

$\ln 3 - 2 \ln x$

* $e^{\ln 3 - \ln x^2}$

$= e^{\ln \frac{3}{x^2}}$

$= e^{\frac{3}{x^2}}$ *

Ex(2) Solve (Find x)

$5 \ln(2x) = 8$

$\Rightarrow \ln(2x) = \frac{8}{5} = 1.6$

take e two sides

$e^{\ln(2x)} = e^{1.6}$

$e = 2x = e^{1.6}$

$x = \frac{e^{1.6}}{2} = 2.477$ *

* Formula :-

$$\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)} .$$

domain $f(x) > 0$

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Ex(1) Write as Single Logarithm

$$5 \ln x - \frac{1}{2} \ln y + 3 \ln z$$

Solution

$$\ln \frac{x^5 \cdot z^3}{\sqrt{y}}$$

Ex(2) (a) $y = \ln(x^2 - 5x + 3) \Rightarrow y' = \frac{2x - 5}{x^2 - 5x + 3}$

(b) $y = \frac{\ln x}{(x+1)} \Rightarrow y' = \frac{(x+1) \cdot \frac{1}{x} - \ln x \cdot (1)}{(x+1)^2}$

$$= \frac{\frac{x+1}{x} - \ln x}{(x+1)^2}$$

Ex:

$y = 3 \sqrt{\frac{(x+3)^2 \cdot (2x^2+4)^4}{(5x+1)^6}}$ Find y'

Solution:

take \ln two sides $\Rightarrow \ln y = \ln 3 \sqrt{\frac{(x+3)^2 \cdot (2x^2+4)^4}{(5x+1)^6}}$

$$\Rightarrow \ln y = \frac{1}{3} [2 \ln(x+3) + 4 \ln(2x^2+4) - 6 \ln(5x+1)]$$

$$\frac{y'}{y} = \frac{1}{3} \left[\frac{2}{x+3} + 4 \cdot \frac{4x}{2x^2+4} - 6 \cdot \frac{5}{5x+1} \right]$$

$$\Rightarrow y' = \frac{1}{3} \left(3 \sqrt{\frac{(x+3)^2 \cdot (2x^2+4)^4}{(5x+1)^6}} \right) \left[\frac{2}{x+3} + \frac{16x}{2x^2+4} - \frac{30}{5x+1} \right]$$

116 ^{chapter (5)}
* Exponential growth and decay model

$$e^{kt} \begin{cases} \text{growth if } k > 0 \\ \text{decay if } k < 0 \end{cases}$$

* The Function $P(t) = P(t_0) \cdot e^{kt}$

satisfies The differential equation | where

(Fast) $P'(t) = kP(t)$ | $P(t_0) = \text{initial Population}$
| $k = \text{constant}$

Ex(1) In April 1994 The Population of a small town was estimated 2500 people. and $A(t) = C e^{kt}$, k is increase rate
IF There Was 3900 in April 1999

Find (a) rate increase (k)?

$$P(t_0) = 2500, P(t) = P(5) = 3900, t = 5 \quad (94 \rightarrow 99)$$

$$\Rightarrow 3900 = 2500 \cdot e^{5k} \Rightarrow e^{5k} = \frac{3900}{2500}$$

$$\Rightarrow e^{5k} = 1.56 \Rightarrow 5k = \ln(1.56) \Rightarrow$$

$$k = \frac{\ln(1.56)}{5} = 0.0889$$

0.0889t

(b) Writ $A(t)$? $\Rightarrow A(t) = 2500 \cdot e^{0.0889t}$

(c) $A(10) \Rightarrow A(10) = 2500 \cdot e^{0.889 \times (10)} = 6087.8$

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Ex(2) (Radioactive decay)

Radium is used in cancer radiotherapy. Let $P(t)$ be the Number of grams of radium after t year.

$P(t)$ satisfy $P'(t) = -0.00043P(t) \rightarrow (\star)$

$$P(0) = 12$$

(a) Find The formula of $P(t)$

$$P(t) = 12 \cdot e^{-0.00043t}$$

(b) initial condition = 12, decay constant = -0.00043

(c) Find The radium after 943 years?

$$t = 943 \Rightarrow P(t) = 12 \cdot e^{0.00043(943)} \approx 8 \text{ grams}$$

(d) how Fast When One gram remains?

نمى 3 (\star)

$$\Rightarrow P(t) = 1 \Rightarrow P'(t) = -0.00043(1) = -0.00043$$

$$\text{i.e rate} = 0.00043$$
